M.A./M.Sc. SEMESTER II (YEAR I)

PAPER-I: ALGEBRA-II

Unit-I: Direct sums of modules, Free modules and vector spaces, Homomorphism extension property, Invariant dimension property, Left exactness of Hom functor, Projective modules.

Unit-II: Injective modules, Baer's characterization, Divisible groups, Existence of enough injectives, Submodules of finitely generated free modules over a PID, Torsion and torsion free modules, p-primary components, Cyclic modules.

Unit-III: Structure theorem for finitely generated modules over a PID, Elementary divisors and invariant factors, Direct sum decomposition of finite abelian groups into cyclic groups and their enumeration.

Unit-IV: Characteristic of a field, Field extensions, Algebraic and transcendental field extensions, Characterization and properties of algebraic extensions, Finitely generated extensions, Simple extensions, Straight edge and compass constructions, Constructible numbers.

Unit-V: Splitting field of a polynomial, Existence and uniqueness of splitting fields, Existence and uniqueness of finite fields, Structure of finite fields, Cycotomic polynomials and extensions, Separable and inseparable extensions, Perfect fields.

Books Recommended:

1. Ramji Lal, Algebra II, Infosys Foundation Series in Mathematical Sciences, Springer, 2017.
2. V. Sahai and V. Bist, Algebra, Narosa Publishing House, 2008.
3. D. S. Dummit and R. M. Foote, Abstract Algebra, Wiley, 2002.
4. J. A. Gallian, Contemporary Abstract Algebra, Cengage India Pv. Ltd., 2019
5. T. W. Hungerford, Algebra, Springer, 1974.
6. Suggested digital platforms: NPTEL/SWAYAM/MOOCS.

PAPER-II: TOPOLOGY

Unit-I: Topological spaces, Metric topology, Open sets, Closed sets, Neighborhoods; Interior, closure, exterior, boundary, and limit points of a set, Dense sets, Separable spaces, Bases and Sub-bases, Subspaces and relative topology..

Unit-II: First and second countable spaces, Lindelöf spaces, Continuous maps and their characterizations, Open and closed maps, Homeomorphisms, Topological property, Product spaces, Projection maps, Convergence of nets and filters.

Unit-III: Quotient spaces and quotient maps, Separation axioms: T_0, T_1,T_2, Regular, T_3, Completely regular, normal, $T_{3\frac{1}{2}}$ and T_4 spaces, Their characterizations and basic properties, Statements of Urysohn's lemma, Tietze extension theorem, and Urysohn's metrization theorem.

Unit-IV: Compact spaces and their characterizations, Sequential and countable compactness, Characterization of compact metric spaces (limit point compactness, sequential compactness, complete and total boundedness), Local compactness, Statement of Tychonoff theorem.

Unit-V: Connected spaces, Connectedness of real line, Connected components, Path connected spaces, Totally disconnected spaces, Locally connected spaces, Locally path connected spaces, Path components, Properties of continuous functions on compact and connected spaces.

Books Recommended:

1. J.R. Munkers, Topology-A First Course, 2nd ed., Prentice Hall of India Pvt. Ltd., New Delhi, 2000.
2. K.D. Joshi, Introduction to General Topology, NewAge International Publishers, New Delhi, 2000.
3. G.F. Simmons, Introduction to Topology and Modern Analysis, McGraw-Hill Education, 2017.
4. T. B. Singh, Elements of Topology, CRC press, New Delhi, 2013.
5. Suggested digital platforms: NPTEL/SWAYAM/MOOCS.

PAPER-III: CLASSICAL MECHANICS

UNIT I: Dynamics of a system of particles:

The linear momentum and the angular momentum, Equations of motion, Conservation of linear and angular momentum, Motion of the centre of mass, Theorems on the rate of change of angular momentum about different points, Kinetic energy of a system of particles relative to the centre of mass of the system.

Rigid bodies as systems of particles, General displacement of a rigid body, Displacement of a rigid body about one of its points and the concept of angular velocity, Computation of the angular velocity of a rigid body in terms of the velocities of two particles of the system chosen appropriately.

UNIT II: Introduction to rigid body motion:

Moments and products of inertia about different axes of uniform rod, Rectangular lamina, Rectangular parallelopiped, Circular wire, Elliptical disc and sphere, Concepts of momental ellipsoid and principal axes.

The angular momentum and kinetic energy of a rigid body in terms of inertia constants, D'Alembert's principle, General equations of motion, Motion of a sphere on horizontal plane, Euler's dynamical equations of motion, Motion under no forces, Invariable line and invariable cone.

UNIT III: Lagrangian Formulation:

Generalized co-ordinates, Geometrical equations, Holonomic and non-holonomic systems, Configuration space, Lagrange's equations using D' Alembert's Principle for a holonomic conservative system, Deduction of equation of energy when the geometrical equations do not contain time *t* explicitly, Lagrange's multipliers case.

UNIT IV: Theory of small oscillations:

Theory of small oscillations, Lagrange's method, normal (principal) co-ordinates and the normal modes of oscillations, Small oscillations under holonomic constraints, Lagrange equations for impulsive motion.

UNIT V: Hamiltonian Formulation:

Generalized momentum and the Hamiltonian for a dynamical system, Hamilton's canonical equations of motion, Hamiltonian as a sum of kinetic and potential energies, Phase space and

Hamilton's variational principle, the principle of least action, Canonical transformations, Poisson-Brackets, Poisson-Jacobi identity, Hamilton-Jacobi theory (outline only).

Books Recommended:

1. Satya Deo and Ramij Rahaman, Classical Mechanics: An Introduction, Narosa Publishing House, New Delhi, 2022.
2. H. Goldstein, Classical Mechanics, Pearson,2011.
3. F. Chorlton, Text Book of Dynamics, CBS Publishers, New Delhi, 1999.
4. Suggested digital platforms: NPTEL/SWAYAM/MOOCS.

PAPER-IV: Any one of the following

PAPER-IV(a): TENSORS AND RIMANNIAN GEOMETRY

Unit I: n-dimensional real vector space, Contravariant vectors, Dual vector space, Covariant vectors, Tensor product, Second order tensors, Tensors of type (r, s), Symmetry and skew symmetry of tensors, Fundamental algebraic operations, Inner product, Quotient law of tensors.

UNIT II: Riemannian metrics, Riemannian manifolds, examples, Affine connections, Covariant differentiation of tensor fields, Covariant derivative along a curve, Parallel transport, Levi-Civita connection, Fundamental Theorem of Riemannian Geometry.

UNIT III: Differential operator on Riemannian manifolds, Gradient vector fields, Divergence of a vector field, Laplacian operator, Lie derivatives of a tensor field with respect to a vector field.

UNIT IV: Riemannian curvature tensor, Identities satisfied by Riemannian curvature tensor, Sectional curvature , Schur's Theorem, Ricci curvature, Scalar curvature, Einstein manifolds, isometries, Notion of covering spaces.

. **UNIT V:** Length of a curve, Riemannian distance function, Geodesics, Local existence and uniqueness for geodesics, Exponential map, Gauss Lemma, Minimizing properties of geodesics, Geodesics normal coordinates.

Books Recommended:

1. R. S. Mishra, A course in Tensors with Application to Riemannian Geometry, Pothishala Pvt. Ltd. Allahabad, 1965.
2. M. P. do Carmo; Riemannian Geometry, Berkhauser, 1992.

3. P. Peterson; Riemannian Geometry, Springer, 2006.
4. J. M. Lee; Riemannian Manifolds: An Introduction to Curvature, Springer, 1997.
5. S. Gallot, D. Hullin. J. Lafontaine; Riemannian Geometry, Springer,3rd edition, 2004.
6. Suggested digital platforms: NPTEL/SWAYAM/MOOCS.

PAPER-IV(b): HYDRODYNAMICS

UNIT I:

Real and ideal fluids, Lagrangian and Eulerian approaches, Convective transport of scalar and vector quantities, Differentiation following the motion and acceleration, Equation of continuity, Velocity potential, Body forces, Surface forces, Stress vector at a point, Nature of stresses, State of stress at a point, Stress tensor, Principal stresses and principal directions, Stress invariants.

UNIT II:

General displacement of a fluid element, Nature of strains, Rates of strain components, Relation between stress and rates of strain, Transformation of stress- components, Transformation of Rates of strain, Euler's equation of motion, Steady motion, Bernoulli's equation.

UNIT III:

Stream lines and vortex lines, Stream tubes and vortex tube, Helmholtz's vorticity theorem, Kelvin's circulation theorem, Energy flux, Mean potential over a spherical surface in a simply connected region, Kinetic energy in irrotational flow, Kelvin's minimum kinetic energy theorem, Uniqueness of the irrotational motion.

UNIT IV:

Two dimensional irrotational motion: Stream function, Complex potential, Concepts of line-sources, sinks, doublets and vortices, Superposition of solutions, The concept of images, The Vortex pair, Vortex rows: Single infinite row of line vortices, the Karman vortex street, Milne-Thomson Circle Theorem, Blasius Theorem, Complex potential for a uniform flow past a circular cylinder, Streaming and circulation about a fixed circular cylinder.

UNIT V:

Three dimensional irrotational flow, Concept of Sources, Sinks and doublets, Axisymmetric flows, Stokes stream function, Statements of Weiss's and Butler's sphere theorems and their applications, Liquid streaming past a stationary sphere, Uniform motion of a sphere in a liquid at

rest at infinity, Gravity waves – Surface waves on the infinite free surface of liquids, Waves at the interface between finitely and infinitely deep liquids.

Books Recommended:

1. L. D. Landau and E. M. Lifshitz, Fluid Mechanics, Butterworth-Heinemann, 2nd Edition, 1987.
2. N. Curle and H. J. Davies, Modern Fluid Dynamics, Vol. I, D. van Nostrand Comp. Ltd., London, 1968.
3. Dr. M.D. Raisinghania, Fluid Dynamics, S. Chand & Company.
4. S. W. Yuan, Foundations of Fluid Mechanics, Prentice-Hall, Englewood Cliffs, NJ, 1967.
5. A. S. Ramsey, A Treatise on Hydrodynamics, Part I, G. Bell and Sons Ltd.
6. Suggested digital platforms: NPTEL/SWAYAM/MOOCS.

PAPER-V: Any one of the following

PAPER-V(a): PROJECT PRESENTATION

PAPER-IV(b): Introduction to Latex