M.A./M.Sc. SEMESTER III (YEAR II)

PAPER-I: MEASURE AND INTEGRATION

Unit-I: Semi-algebra, Algebra, Monotone class, σ- algebra, Measure and outer measure, Measure spaces, Borel sets, Lebesgue outer measure on $\mathbb{R}$ and its properties, σ- algebra of Lebesgue measurable sets, Lebesgue measure space, Translation invariance of Lebesgue measure.

Unit-II: Outer and inner approximation of Lebesgue measurable sets, Vitali's theorem, The Cantor ternary set and Cantor-Lebesgue function, Continuity of measure, Caratheodory extension process of extending a measure on a semi-algebra to generated σ- algebra, Completion of a measure space.

Unit-III: Measurable functions on a measure space and their properties, Borel and Lebesgue measurable functions, Littlewood's three principle (statement only), Simple approximation Theorem, Simple functions and their integrals,

Unit-IV: Lebesgue integral on $\mathbb{R}$ and its properties, Comparison of Riemann integral and Lebesgue integral, Bounded convergence theorem, Integration of nonnegative measurable functions on a measure space, Fatou's lemma, Monotone convergence theorem, Integration of general measurable functions, Continuity of integration, Lebesgue dominated convergence theorem.

Unit-V: Inequalities of Young, Hölder and Minkowski, Normed Linear spaces, Riesz-Fischer theorem, Measure and integration on product spaces, Fubini's theorem, Lebesgue measure on $\mathbb{R}^2$ and its properties.

Books Recommended:

1. H.L. Royden, and P. M. Fitzpatrick, , Real Analysis, 4th Edition, Pearson, 2010.
2. I. K. Rana, An Introduction to Measure and Integration, 2nd edition, Narosa Publishing House India, 2000.
3. P. K. Jain and V. P. Gupta, Lebesgue Measure and Integration, New Age International (P) Limited, New Delhi, 1986.
4. Suggested digital platforms: NPTEL/SWAYAM/MOOCS.

PAPER-II: ORDINARY DIFFERENTIAL EQUATIONS AND BOUNDARY VALUE PROBLEMS

UNIT I:

Initial value problems, Picard's iterations, Lipschitz conditions, Sufficient conditions for being Lipschitzian in terms of partial derivatives, Picard's theorem for local existence and uniqueness of solutions of an initial value problem of first order which is solved for the derivative, Uniqueness of solutions with a given slope, Singular solutions, *p*- and *c*-discriminant equations of a differential equation and its family of solutions respectively, Envelopes of one parameter family of curves, Singular solutions as envelopes of families of solution curves, Sufficient conditions for existence and nonexistence of singular solutions, examples.

UNIT II:

Linear independence and Wronskians, General solutions covering all solutions for homogeneous and non-homogeneous linear systems, Abel's formula, Method of variation of parameters for particular solutions, Linear systems with constant coefficients, Matrix methods, Different cases involving diagonalizable and non-diagonalizable coefficient matrices, Real solutions of systems with complex eigenvalues.

UNIT III:

Convergence of real power series, Radius and interval of convergence, Ordinary and singular points, Power series solutions, Frobenius' generalized power series method, Indicial equation, different cases involving roots of the indicial equation, Regular and logarithmic solutions near regular singular points.

UNIT IV:

Legendre's equation, Solution by power series method, polynomial solution, Legendre polynomial, Rodrigues' formula, Generating function, Recurrence relations, Orthogonality relations, Fourier-Legendre expansion, Bessel's equation, Bessel functions of I and II kind, Recurrence relations, Bessel functions of half-integral orders, Sturm comparison theorem, Zeros of Bessel functions, Orthogonality relations, Generating function.

UNIT V:

Boundary-value problems: Orthogonal and orthonormal sets of functions, Sturm-Liouville (S-L) problems, Eigenvalues and Eigenfunctions of S-L problems , Reality of eigenvalues and orthogonality of eigenfunctions of S-L problems, Singular Sturm-Liouville problems, Green's function, Construction of Green functions, Solution of Boundary value problem by Green's function.

Books Recommended:

1. B. Rai, D. P. Choudhury and H. I. Freedman, A Course in Ordinary Differential Equations, Narosa Publishing House, New Delhi, 2002.
2. E. Kreyszig, Advanced Engineering Mathematics, Wiley India Pvt. Ltd., 8thEdition, 2001.
3. S.G. Deo, V. Raghavendra, R. Kar and V. Lakshmikanthanm, text Book of Ordinary Differential equations, McGraw Hill Education, 2017.
4. E. A. Coddington, An Introduction to Ordinary Differential Equations, Prentice Hall of India, New Delhi, 1968.
5. Suggested digital platforms: NPTEL/SWAYAM/MOOCS.

PAPER-III: DIFFERENTIAL GEOMETRY OF MANIFOLDS

UNIT I: Topological manifolds, Compatible charts, Smooth manifolds, examples, Smooth maps and diffeomorphisms, Definition of a Lie group, examples.

UNIT II: Tangent and cotangent spaces to a manifold, Derivative of a smooth map, Immersions and submersions, submanifolds , Vector fields, Algebra of vector fields, φ-related vector fields, Left and right invariant vector fields on Lie groups.

UNIT III: Integral curves of smooth vector fields, Complete vector fields, Flow of a vector field, Distributions, Tensor fields on manifolds, r-forms, Exterior product, Exterior differentiation, Pull-back differential forms.

UNIT IV: Affine connections (covariant differentiation) on a smooth manifold, Torsion and curvature tensors of an affine connection, Identities satisfied by curvature tensor.

UNIT V: Riemannian metrics, Riemannian manifolds, Submanifolds, Local isometry and isometry, Levi-Civita connection, Fundamental Theorem of Riemannian Geometry, Riemannian curvature tansor, Identities satisfied by Riemannian curvature tensor, Ricci tensor, Scalar curvature, Sectional curvature of Riemannian manifolds.

Books Recommended:

1. S. Kumaresan; A course in Differential Geometry and Lie groups, Hindustan Book Agency, 2002.

2. U. C. De, A. A. Sheikh; Differential Geometry of Manifolds, Narosa Publishing House, 2007.
3. W. M. Boothby; An Introduction to Differentiable Manifolds and Riemannian Geometry, Academic Press, revised, 2003.
4. T. J. Willmore; Riemannian geometry, Oxford Science Publication, 1993.
5. Suggested digital platforms: NPTEL/SWAYAM/MOOCS.

PAPER-IV: Any one of the following:

PAPER-IV(a): MATHEMATICAL MODELLING

Unit I: Simple situations requiring mathematical modelling, Techniques of mathematical modelling, Classifications, Characteristics and limitations of mathematical models, Some simple illustrations.

Unit II: Mathematical modelling through differential equations: Overview of basic concepts in ODE and stability of solutions, Critical points and their local and global stability, Stability by variational matrix method and Liapunov's direct method.

Unit III: Linear growth and decay models, Nonlinear growth and decay models, Mathematical modelling in dynamics through ordinary differential equations of first order, Single species population model: The exponential model and the Logistic model.

Unit IV: Biological interactions, Models for interacting species, Lotka-Volterra predation models, Dynamics of Simple Lotka-Volterra model, Role of density dependence in the prey, Analysis of a Predator-Prey Model with limit cycle periodic behaviuor.

Unit V: Competition: Lotka-Volterra models, Extension to Lotka-Volterra models, Competition for fixed resources and renewable resources, Models for Mutualism, Obligate and non-obligate mutualism.

Books Recommended:

1. J. N. Kapur, Mathematical Modeling, New Age International,1988.
2. J. D. Murray, Mathematical Biology: An Introduction, Springer, 2002 .
3. H.I. Freedman, Deterministic Mathematical Models in Population Ecology, Marcel-Dekker, New Yark, 1980 .

4. G, F, Simmons, Differential Equations with Applications and Historical Notes, Tata-McGraw Hill 1991.

5. Suggested digital platforms: NPTEL/SWAYAM/MOOCS.

PAPER-IV(b): ADVANCED LINEAR ALGEBRA

Unit-I: Algebraic and geometric multiplicities of eigenvalues, Invariant subspaces, Minimal polynomial, Triangulable linear operators, Characterizations of diagonalizable linear operators and matrices in terms of multiplicities and minimal polynomial.

Unit-II: Simultaneous triangulation and diagonalization, Invariant direct sum decompositions, Primary decomposition theorem, Characterization of diagonalizability in terms of projections, Diagonalizable and nilpotent parts of an operator.

Unit-III: Cyclic subspaces and annihilators, T-annihilator, cyclic decompositions and rational canonical form.

Unit-IV: The Jordan canonical form, Symmetric bilinear forms, Diagonalization of symmetric matrices, Sylvester's law of inertia.

Unit-V: Positive definite matrices and polar decomposition, QR, LU and Cholesky decomposition of matrices, Singular value decomposition.

Books Recommended:

1. K. Hoffman and R. Kunze, Linear Algebra, PHI, 2015.
2. V. Sahai and V. Bist, LinearAlgebra, Narosa Publishing House, 2013.
3. Ramji Lal, Algebra II, Infosys Foundation Series in Mathematical Sciences, Springer, 2017.
4. S.H. Friedberg, A. J. Insel and L. E. Spence, Linear Algebra, PHI, 2003.
5. Suggested digital platforms: NPTEL/SWAYAM/MOOCS.

PAPER-V: Any one of the following:

PAPER-V(a): Project Presentation Based On Survey/ Seminar/ Assignment

PAPER-IV(b): Introduction to SCILAB/MATLAB