M.A./M.Sc. SEMESTER IV (YEAR II)

PAPER-I: FUNCTIONAL ANALYSIS

Unit-I: Normed Linear spaces, Equivalent norms, Complete metric spaces, Banach Spaces, Examples and their topological properties, Finite dimensional normed linear spaces, Holder's and Minkowski's inequalities, l^p-spaces, L^p-spaces, Subspace and quotient space of Banach spaces.

Unit-II: Continuity and Convergence, Continuous linear transformations, Bounded linear transformations, Normed linear space of bounded linear transformations, Baire category theorem, Linear functionals, Hahn-Banach theorem,

Unit-III: Natural embedding of N into N^{**}, Conjugate spaces, Reflexivity, Weak and Weak*-topology on a conjugate space, Open mapping theorem, The Closed graph theorem, Uniform boundedness theorem, Conjugate of an operator.

Unit-IV: Inner product spaces, Cauchy-Schwarz inequality, Hilbert Spaces, Orthogonal complements, Orthonormal sets, Complete orthonormal sets, Bessel's inequality, Fourier expansion, Parseval's identity, Gram Schmidt orthogonalization process.

Unit-V: Riesz representation theorem, Reflexivity of Hilbert spaces, Adjoint of an operator, Self adjoint operators, Normal operators, Unitary operators, Projections, Spectrum of an operator, Spectral theorem for a normal operator on a finite dimensional Hilbert space.

Books Recommended:

1. S. Ponnusamy, Foundation of Functional Analysis, Narosa Publishing House, New Delhi,2002.
2. G.F. Simmons, Introduction to Topology and Modern Analysis, McGraw-Hill Education, 2017.
3. E. Kreyszig: Introductory Functional Analysis with Applications: Wiley student Edition 2007.
4. J. B. Conway: A course in Functional Analysis, Springer: Second Edition, 2007.
5. Suggested digital platforms: NPTEL/SWAYAM/MOOCS.

PAPER-II: ADVANCED FLUID MECHANICS

UNIT-I: Stress Principle of Cauchy, Equations for conservation of linear and angular Momentum, Constitutive equations for Newtonian fluids, Newton's law of viscosity, Navier-Stokes equations in Vector and Tensor forms, Navier- Stokes equations in orthogonal coordinate systems (particularly in Cartesian, cylindrical and spherical coordinate systems).

UNIT-II: Vorticity equations; Energy dissipation due to viscosity, Dynamical similarity and dimensionless numbers and their significance in the fluid dynamics, Some exact solutions –Fully developed plane Poiseuille and Couette flows between parallel plates, Steady flow between pipes of uniform cross-section.

UNIT-III: Couette flow between coaxial rotating cylinders, Small Reynolds number flow – Flow between steadily rotating spheres, Stokes equations, Dynamic equation satisfied by stream function, Relation between pressure and stream function; General stream function solution of Stokes equations in spherical polar coordinates; Steady flow past a sphere, Drag on a body.

UNIT-IV: Flow past a circular cylinder, Stokes paradox, Boundary layer concept, Two-dimensional boundary layer equations, Boundary layer on a semi-infinite plane, Blasius equation and solution, Karman's Integral method, Displacement thickness, Momentum thickness and Energy thickness.

Unit-V: Nature of Magnetohydrodynamics, Main assumptions of MHD, Basic equations of non- viscous and viscous magnetohydrodynamics: mass, momentum and energy conservation laws. Basic Properties of the magnetic field and MHD terms: Magnetic Reynolds number, magnetic viscosity, magnetic pressure, magnetic diffusion and frozen- in- effect. Magnetohydrodynamic boundary conditions, Magnetohydrodynamic Flows, Formulation and solution of Linear flow.

Reference books:

1. Z.U.A. Warsi, Fluid Dynamics, CRC Press (2005)
2. J. Happel and H. Brenner, Low Reynolds Number Hydrodynamics, Kluwer Academic Publishers group, (1983).

3. T.C. Papanastasiou, G.C.Georgiou, A.N.Alexandrou, Viscous Fluid Flow; CRC Press (2000).
4. L. D. Landau and E. M. Lifshitz, Classical Electrodynamics, Butterworth-Heinemann, 2nd Edition, 1984.
5. A. Jaffery, Magnetohydrodynamics, Oliver and Boyd, N.Y. 1966.
6. Suggested digital platforms: NPTEL/SWAYAM/MOOCS.

PAPER-IV: Any one of the following

PAPER-IV(a): WAVELETS

UNIT I: The discrete Fourier transform and the inverse discrete Fourier transform, their basic properties and computations, The fast Fourier transform, Construction of wavelets on Z_N, First stage and by iteration, The Haar system, Shannon wavelets, Daubechies' D6 wavelets on Z_N, Description of $l^2(Z)$, $L^2[-\pi,\pi]$, $L^2(R)$, Their orthonormal bases,

UNIT II: Fourier transform and convolution on $l^2(Z)$, wavelets on Z ,Haar wavelets on Z, Daubechies' D6 wavelets for $l^2(Z)$.Orthonormal bases generated by a single function in $L^2(R)$, Fourier transform and inverse Fourier transform of a function f in $L^1(R)\cap L^2(R)$, Parseval's relation, Plancherel's formula, Orthonormal wavelets in $L^2(R)$, Balian-Low theorem.

UNIT III: Multi-resolution analysis and MRA wavelets, certain function in $L^2(R)$ for which $\{\psi_j, k\}$ does not form an orthonormal system, compactly supported wavelets, Band-limited wavelets.

UNIT IV: Franklin wavelets on R, Characterization of MRA wavelets (Sketch of the proof), Minimally Supported Wavelets, Wavelet Sets, Characterization of two-interval wavelet sets, Shannon wavelet, Journe's wavelet.

Books Recommended:

1. Michael W. Frazier, An Introduction to Wavelets through Linear Algebra, Springer Verlag, 1999.
2. Eugenio Hernández and Guido Weiss, A First Course on Wavelets, CRC Press, 1996.
3. G. Kaiser, A Friendly Guide to Wavelets, Birkhauser, 1994.

4. C. K. Chui, An Introduction to Wavelets, Academic Press, 1992.
5. Suggested digital platforms: NPTEL/SWAYAM/MOOCS.

PAPER-IV(b): REPRESENTATION THEORY OF FINITE GROUPS

Unit-I: Simple modules, Schur's lemma, Semi-simple rings and modules, Wedderburn structure theorem for semi-simple modules and rings.

Unit-II: Group algebras, Maschke's theorem, Linear and matrix representation of a finte group, Representations of G and k(G)-modules, Equivalent representations, Decomposition of regular representation, Number of irreducible representation.

Unit-III: Characters, Irreducible characters, Orthogonality relations, Integrality properties of character, Character ring, Burnside's $p^a q^b$ theorem.

Unit-IV: Representations of direct product of two groups, Induced representation, The character of an induced representation, Frobenius reciprocity theorem, Construction of irreducible representation of D_n, A_4, S_4 and S_5.

Books Recommended:

1. Ramji Lal, Algebra II, Infosys Foundation Series in Mathematical Sciences, Springer, 2017.
2. D. S. Dummit and R. M. Foote, Abstract Algebra, Wiley, 2002.
3. J. P. Serre, Linear Representations of finite Groups, GTM-42, Springer, 1977.
4. L. Dornhoff, Group Representation Theory , Part A, Academic Press, 1971.

PAPER-IV(c): ALGEBRAIC NUMBER THEORY

Unit-I: Algebraic number fields, Algebraic numbers and algebraic integers, Ring of algebraic integers, Calculation for quadratic, cubic and cyclotomic case.

Unit-II: Norm and traces, Integral bases and discriminants, Ideals in O_K, Dedekind domains, Fractional ideals and unique factorization.

Unit-III: Dedekind's theorem, Factorization in O_K, The ideal class group, Lattices in $\mathbb{R}^n$, Minkowaski's bound, Finiteness of ideal class group.

Unit-IV: Exponents of ideal class groups. Dirichlet's unit theorem, Units in real quadratic real fields and cyclotomic fields.

Books Recommended:

1. J. Esmonde and M. R. Murty, Problems in Algebraic Number Theory, Springer, 1999.
2. J. Marcus, Number Fields, Springer, 1977.
3. K. Ireland and M. Rosen, A Classical Introduction to Modern Number Theory, ,GTM-84, Springer, 1990.

PAPER-IV(d): SPECIAL FUNCTIONS

Unit-I: The Gamma Function**:** Analytic Character, Tannery's theorem, Euler's limit formula, Duplication formula, Eulerian integral of the first kind, Euler's Constant, Canonical product, Asymptotic expansions, Watson's lemma, Hankel's contour integral.

Unit-II: The Hypergeometric Function: An Integral representation, Its differential equation and solutions, F(a, b, c; 1) as a function of the parameters, evaluation of F(a, b, c; 1), contiguous function relations, the hypergeometric differential equation, logarithmic solutions of the hypergeometric equation.

Unit-III: F(a, b, c; z) as a function of its parameters, Elementary series manipulations, Simple transformations, relations between functions of (z) and, (1-z) quadratic transformations, theorem, due to Kummer, additional properties, Confluent hypergeometric function: Basic properties of ${}_1F_1$, Kummers first formula, Kummers second formula.

Unit-IV: Generalized hypergeometric series: The function pFq, the exponential and bionomial functions, differential equation, contiguous function relations, integral representation pFq, with unit argument, Saalshutz, theorem, Whipples theorem, Dixons theorem, Barnes's contour integral of F(a, b, c; z).

Books Recommended:

1. N.N. Lebedev and R. A. Silverman, Special Functions and their applications, Dover Publication Inc.1972.
2. A. Chakrabarti, Elements of Ordinary Differential Equations and Special Functions, New Age International Publisher, 1996.
3. Earl. D. Ranvillie, Special Functions, Macmillan, 1960.

PAPER-IV(e): GALOIS THEORY

Unit-I: Field extensions, Algebraic extensions, Splitting field of a polynomial, Separable and inseparable extensions, Primitive element theorem.

Unit-II: Automorphism of fields, Dedekind's theorem, Fixed fields, Normal extensions, Splitting fields and normality, Normal closures.

Unit-III: Galois extensions, Fundamental theorem of Galois theory, Computation of Galois groups of polynomials.

Unit-IV: Cyclic extensions, Radical extensions, Galois' criterion for solvability of polynomial by radical operations, Abel-Ruffini theorem.

Books Recommended:

1. Ramji Lal, Algebra II, Infosys Foundation Series in Mathematical Sciences, Springer, 2017.
2. V. Sahai and V. Bist, Algebra, Narosa Publishing House, 2008.
3. D. S. Dummit and R. M. Foote, Abstract Algebra, Wiley, 2002.
4. P. Morandi, Field and Galois Theory, springer 1996.

PAPER-IV: MAJOR RESEARCH PROJECT/DESSERTATION